A semi-empirical approach to cutting force prediction for point-attack picks

by R.M. Goktan* and N. Gunes*

Synopsis

Point-attack picks are one of the most widely used tools for the mechanical excavation of rock and coal. In the literature, some practical formulas have been developed to estimate their performances under varying cutting conditions such as the pick geometry, cutting depth and cutting geometry. In this study, it is shown that Evans’s theoretical model, the most cited in the literature, has serious limitations concerning the changes in the cutting geometry. Analysis of the two independent full-scale linear rock cutting experimental data reveals that both his theory and its recent modification can not fully explain the mechanism of rock breakage under asymmetrical attack. The cutting force predictions made by employing such models are shown to be too significantly weak to be of any service in practice.

In this study, a semi-empirical approach was followed by means of which cutting forces under varying cutting geometries could be made. By analysing the full-scale rock cutting test data, prediction equations of the peak cutting force and mean cutting force have been developed. Comparisons of laboratory determined and predicted force values indicate that the suggested prediction equations are applicable to quite different rock materialism which have uniaxial compressive strength in the range of about 30–170 MPa, and valid for different cutting geometries. It is also shown by regression analysis that the established prediction equations are statistically significant.

Introduction

In the mining and civil engineering industries point-attack picks (conical picks) are extensively employed on mechanical excavators such as roadheaders, continuous miners and longwall shearsers to cut relatively harder rocks/coal compared with radial picks. They are more durable than radial picks, and can economically cut intact rocks having up to around 120 MPa uniaxial compressive strength at low abrasivity index1.

A review of the literature shows that the general behaviour of point-attack picks in rock cutting2-3, wear mechanism4-7, efficiency and performance8-12, measurement of pick forces for machine design13, comparison with radial picks14, rotational properties15, and chip formation mechanisms16,17 are among the topics that have received the attention of researchers.

Another topic that has also received attention is the prediction of cutting forces. Knowing the magnitude of the cutting forces is an important aspect of machine design, since it allows the engineers to estimate the cutterhead torque and machine power requirements for a particular application. Perhaps due to the complexity of the required three-dimensional analysis of their cutting mechanism, only limited analytical models have been proposed for the prediction of cutting forces. Evans’s rock cutting theory18 for point-attack picks is the most widely recognized and most frequently cited in the literature. His theory attempts to give an insight into the mechanism of rock breakage under the action of a symmetrically acting point-attack pick, and enables the peak cutting force to be calculated from pick geometry, rock properties and cutting depth. Although it is one of the most practical formulas offered for calculating the peak cutting force, as emphasized by Evans and some other authors1,9, it has some deficiencies, which have recently been improved by modifications to the theory19.

Both in the original and modified theories of Evans, the cutting pick is assumed to act symmetrically along the line of advance, but in practice cutting picks do not act symmetrically on the cutter heads of mechanical excavators. This could be an important point in the calculation of cutting forces. In this work, considering the current limitations of these analytical models, Evans’s modified rock cutting theory19 is empirically extended by analysing full-scale rock cutting test data. Valid for asymmetrical cutting conditions, semi-empirical equations are developed for the prediction of peak cutting force and mean cutting force in terms of pick geometry, cutting geometry, cutting depth and tensile strength of rock. Regression analysis has also been carried out to check the validity of the proposed prediction equations.

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Outlines of the analytical models

The cutting forces acting on the pick determines the suitability and applicability of a pick for a particular application. Magnitudes of the cutting forces are also required for determining machine specifications and designing the cutterheads of mechanical excavators. Cutting forces (Figure 1) may be measured by full-scale cutting tests in the laboratory, or estimated by using formulas developed analytically or empirically.

Evans’s theory for point-attack picks attempts to enable the engineers to estimate the peak cutting force for a given rock, when direct measurement of the cutting force is not available. His theory is based on the assumption that the penetration of a conical pick attacking a buttock of rock (Figure 2a) produces radial compressive stresses in the rock, accompanied by tensile hoop stresses (Figure 2b). Tensile cracks will open up at the interface between pick and rock when the stress equals the tensile strength of the rock.

The full theory is given elsewhere and will not be repeated here.

The prediction formula provided by Evans is as follows:

\[
FC = \frac{16\pi \sigma_t^2 d^2}{\sigma_c \cos^2 \theta}
\]  

where

- FC = Peak cutting force
- \( \sigma_t \) = Tensile strength of rock
- d = Cutting depth
- \( \sigma_c \) = Uniaxial compressive strength of rock
- \( \theta \) = Semi-angle of conical pick (degrees)

As can be followed from above, there are some deficiencies of Equation [1]. These are: (i) The cutting force (FC) does not reduce to zero when \( \theta = 0^\circ \), although it should, and (ii) the cutting force is inversely proportional to the compressive strength of rock, which is not the case in practice. The general features of the original theory being kept, these two shortcomings of Evans’s theory were eliminated in a recent study by Goktan where a modified prediction equation was proposed:

\[
FC = \frac{4\pi \sigma_t d^2 \sin^2(\theta + \psi)}{\cos(\theta + \psi)}
\]  

where the parameters FC, \( \alpha \), d, and \( \theta \) are as defined earlier in Equation [1], and \( \psi \) is the friction angle between the pick and rock (degrees). The detailed mathematics of the modified theory will not be repeated here, but from Equation [2] it can be followed that (i) one of the shortcomings of Evans’s theory (that FC does not reduce to zero when \( \theta = 0^\circ \)) is eliminated, and (ii) the problematic parameter uniaxial compressive strength does not appear in the prediction equation.

Despite this improvement brought to the original Evans’s theory, the model adopted so far is still that of a point-attack pick symmetrical about the line of advance (Figure 2a), but the cutting action of practical picks is essentially asymmetrical since they are mounted on a cutterhead with a forward angle of attack (\( \gamma \)) (Figure 1). Therefore, an attempt is made here to take account of asymmetrical attack by introducing the parameter ‘rake angle’ (\( \alpha \)), which is the angle, in the plane of motion, between the front face of the pick and a line drawn normal to the cutting direction from the leading tip of the pick (Figure 1).

Semi-empirical approach for asymmetrical attack

The cutting force prediction abilities of Equations [1] and [2] were checked by data obtained from full-scale rock cutting experiments. The raw data of the experimental works was referenced to two recent PhD studies conducted at the Rock Cutting Research Centre of Istanbul Technical University Mining Engineering Department.

A full-scale linear cutting rig (Figure 3) accommodating Sandvik S35/80H and Board U47 HD50 type commercial point-attack picks was used at different cutting geometries throughout the experiments. The rig used for the full-scale tests has been developed under the NATO-TU-Excavation Project in Istanbul Technical University, and can
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![Figure 2](image1.png)  
**Figure 2**—Illustrating assumptions of Evans's tensile breakage model (a) Pick acting symmetrically on a buttock of rock (b) View along the direction of cut

![Figure 3](image2.png)  
**Figure 3**—The general view of the full-scale cutting rig

To accommodate rock samples 70·50·50 cm in size. The experiments were conducted at unrelieved cutting conditions at depths which were previously determined to be near optimum conditions with regards to specific energy. The tested materials have uniaxial compressive strengths ranging from 30 MPa to 174 MPa, including rocks and ores. An abbreviated description of the referenced test conditions, the corresponding mechanical properties of the tested rocks, and measured and predicted peak cutting force values are presented in Table I.

As can be followed from rock cutting test data presented in Table I, Evans's theory and its modification are not consistent with full-scale laboratory cutting experiments, and prediction success of both models are too weak to be of any service in practice. The magnitudes of the peak cutting force values (FC) predicted by using these two models are significantly lower than that of the measured values. It is felt by the present authors that this fact might originate from the assumptions made in both of the models, where only the symmetrical cutting conditions have been considered. It is well established in the literature that changes in the cutting geometry (i.e. rake angle or angle of attack) significantly govern the performance of picks due to the changes in stress distributions, hence influencing obtained force values.

Another limitation of the analytical models is that the shape of the cutting pick is assumed to be an exact cone, although in practice no commercial pick has the geometry of an exact cone. From the point of view of manufacture and practical use, there is always a certain amount of rounding near the tip of the pick. The rounding of the tip can significantly affect the cutting forces, especially at small and inefficient depths of cut.
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### Table I

<table>
<thead>
<tr>
<th>Rock Type</th>
<th>Tensile strength MPa</th>
<th>Uniaxial compressive strength MPa</th>
<th>Cutting depth mm</th>
<th>Calculated peak cutting force FC</th>
<th>Calculated peak cutting force FC (Goktan19) kN</th>
<th>Measured peak cutting force, FC kN</th>
</tr>
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<td>6.72</td>
<td>6.53</td>
<td>25.62</td>
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</table>

* Cone angle: 80° ; Angle of attack : 57°; Rake angle: (-7°); Clearance angle: 15°
**Cone angle: 105° ; Angle of attack : 57°; Rake angle: (-19.5°); Clearance angle: 4.5°
***Cone angle: 80° ; Angle of attack : 55°; Rake angle: (-5°); Clearance angle: 15°

Test data from 1 to 6 were referred to Kel22, and 7 to 14 were referred to Tuncdemir24

Considering the practical limitations of the mentioned analytical models, it is possible to draw the important conclusion that they do not serve as a reliable guide for cutting force prediction purposes. This could be important from a practical point of view where it has been suggested by some researchers that the cutting force values predicted by such models could be used for calculating the torque and power requirements of roadheaders when direct measurement of the cutting force is not available.

From the above discussion it can be deduced that, perhaps due to the three-dimensional modelling difficulties and its complex mechanism, theoretical aspects of rock cutting with point-attack picks have not been yet fully established, and further research should be carried out in this field of rock cutting mechanics. In such problems of rock engineering where analytical solutions are not possible due to complexities involved, it is a common practice to make use of empirical formulas. To achieve this goal, the present authors have examined the predictions of point-attack picks under asymmetrical attack. An analysis of the test data, it has been found that a reasonably close fit to experimental results could be obtained by:

\[ FC = \frac{12 \pi \alpha_1 \sigma_d^2 \sin \left(\frac{1}{2}(90 - \alpha) + \psi\right)}{\cos \left(\frac{1}{2}(90 - \alpha) + \psi\right)} \tag{3} \]

where
- \( FC \) = Peak cutting force
- \( \alpha_1 \) = Tensile strength of rock
- \( d \) = Cutting depth
- \( \theta \) = Semi-angle of conical pick (degrees)
- \( \alpha \) = Rake angle (degrees)
- \( \psi \) = Friction angle between the pick and rock (10°)

It is important to note that the effect of cutting geometry on the cutting force of point-attack picks is reflected in Equation [3] by the inclusion of rake angle (\( \alpha \)). This point has been neglected in the analytical models proposed in the literature.

A close examination of Table II and Figure 4 indicates that predictions based on Equation [3] may be regarded as satisfactory for this field of rock excavation where not only the trends are confirmed, but also the magnitudes of peak cutting forces (FC). Calculated FC values from Equation [3] are in better agreement with the measured data, more significantly in magnitude, than those calculated by using the prediction equations provided by Evans18 and Goktan19.

### Estimation of mean cutting forces

The cutting force derived from Equation [3] is the value occurring at the instant of rock failure, and therefore it is the peak cutting force (FC). However, in computing the cutterhead torque and power requirements of a machine, it is the ‘mean cutting force’ (FC') values which are used.

The mean cutting force is one of the most important parameters in the general field of rock cutting mechanics. Considering this fact, in this study an attempt was also made to develop a prediction equation of the mean cutting force.

Analysis of the full-scale rock cutting experimental data made by the present authors has shown that there exists a strongly correlated relationship (Figure 5) between FC and FC' in the form:

\[ FC / FC' = 3 \tag{4} \]

It is interesting to note that the existence of such a relationship between FC and FC' values is also supported by the works of Ranman17 who has also suggested the same figure for this ratio. Therefore, upon combining Equation [3]
and Equation [4], it is possible to establish a practical prediction equation of the mean cutting force (FC') in the form:

\[ FC' = 4\pi \sigma_c d^2 \sin^2 \left( \frac{1}{2}(90 - \alpha + \psi) \right) \cos \left( \frac{1}{2}(90 - \alpha + \psi) \right) \]  

[5]

with the same notations as quoted in Equation [3].

Although Equation [5] was empirically derived to fit the experimental data, in order to check its predictive value, FC' values calculated from Equation [5] were compared with the laboratory measured FC' values. As can be followed from Figure 6, the force values calculated by using Equation [5] show very close correlation with the actual values measured in the laboratory.

Validation of the proposed prediction equation was checked by regression analysis (Table III). The output shows the results of fitting a linear model to describe the relationship between measured FC' and calculated FC'. The equation of the fitted model is:

\[ FC' = 0.4232 + 0.9599 \times \text{Calculated FC'} \]

[6]

The R-Squared statistic indicates that the model as fitted explains 88.5 per cent of the variability in calculated FC'. The correlation coefficient equals 0.94, indicating a strong relationship between the variables. The standard error of the estimate shows the standard deviation of the residuals to be 1.81. Since the P-value in the ANOVA table is less than 0.01, there is a statistically significant relationship between measured FC' and calculated FC' at the 99 per cent confidence level. Therefore, it is concluded that the model is statistically valid.

Conclusions

Although full-scale laboratory rock cutting tests can give the basic data of relationships between cutting forces and rock properties, the main shortcomings of these tests is that the apparatus and procedures are complicated, and require highly experienced personnel. In cases where full-scale laboratory rock cutting facilities are not available, analytical or empirical methods may be employed for the estimation of cutting forces.

Of the analytical models, perhaps the most acceptable model for rock cutting with point-attack picks is that of Evans. However, evaluation of data obtained from two independent full-scale laboratory rock cutting experiments indicate that the prediction abilities of both the original and modified analytical models of Evans do not serve as a reliable guide for the prediction of cutting forces under asymmetrical cutting conditions. It should be appreciated that the heterogeneity of the rocks and the three-dimensional action of the point-attack picks require a complex mathematical treatment, which makes pick performance prediction a complicated task.

Table II

<table>
<thead>
<tr>
<th>Rock type</th>
<th>Tensile strength MPa</th>
<th>Uniaxial compressive strength MPa</th>
<th>Cutting depth mm</th>
<th>Measured peak cutting force (FC) kN</th>
<th>Calculated peak cutting force (FC') kN</th>
<th>Measured mean cutting force (This work) FC kN</th>
<th>Calculated mean cutting force (FC') kN</th>
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* Cone angle: 80°; Angle of attack: 57°; Rake angle : (- 7°); Clearance angle : 17°
**Cone angle: 105°; Angle of attack: 57°; Rake angle : (- 19.5°); Clearance angle : 4.5°
***Cone angle: 80°; Angle of attack: 55°; Rake angle : (-5°); Clearance angle : 15°

Test data from 1 to 6 were referred to Kel22, and 7 to 14 were referred to Tuncdemir21

Figure 4—Relationship between the calculated and measured peak cutting forces

\[ y = 0.8137x + 3.5053 \quad R^2 = 0.8292 \]
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Figure 5—Relationship between the measured peak and mean cutting forces

\[ y = 0.3743x - 0.4544 \]
\[ R^2 = 0.968 \]

Figure 6—Relationship between the calculated and measured mean cutting forces

\[ y = 0.96x + 0.422 \]
\[ R^2 = 0.8852 \]

Table III

<table>
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<th>Statistical results of the regression analysis</th>
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Analysis of variance

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Correlation coefficient = 0.94
R-squared = 88.51
Standard error of estimate = 1.81
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Due to the difficulties of three-dimensional modelling of asymmetrically acting point-attack picks, approaches to this problem are essentially empirical. In this work, based on the results of full-scale linear rock cutting tests, semi-empirical prediction equations for the calculation of peak and mean cutting forces have been developed. Comparisons of laboratory determined and calculated force values of different rock materials indicate that the suggested prediction equations may be applicable to rock materials which have uniaxial compressive strength in the range of about 50–170 MPa, under different cutting geometries.

The agreement between the measured and calculated force values are as good as one can expect in this field of rock excavation and are statistically well correlated. It may be interesting to note that the established prediction equations are applicable to quite different materials like rocks and ores. Industrial significance of this work is that the presented empirical equations may be used by the engineers, at least for preliminary calculations, for the estimation of machine power and cutterhead torque requirements when direct force measurements are not available. Finally, it is suggested that the general applicability of the proposed prediction equations be checked and updated for other rock types by further studies.

Acknowledgement

The authors wish to thank Prof N. Bilgin (Istanbul Technical University) and Prof N. Akcin (Zonguldak Karaelmas University) for kindly providing the data for full-scale rock cutting experiments.

References
