Development of a gravity flow numerical model for the evaluation of drawpoint spacing for block/panel caving

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Transaction Paper

Synopsis

Block caving methods, when operated under favourable rock mass conditions, can achieve higher production rates and lower operating costs than other various existing underground mining applications. For this reason, it has been considered the preferred method for the mining of deep and large orebodies in current and future operations around the world. One of the key aspects in the design of block caves is the selection of the production level layout, which, among other parameters, is based on the gravity flow characteristics of the caved rock. This is because gravity flow has a large impact on total ore recovery and the amount of dilution in a caving operation. Today, there are a number of computer based methods which aim to emulate the gravity flow pattern of the caved rock.

In this paper, the authors present the development of FlowSim an improved model of gravity flow based on the cellular automaton approach, to estimate dilution entry, mixing and ore recovery. As part of the model development, flow simulations were conducted and subsequently compared to full-scale data collected at two mine sectors operated by Codelco Chile. The results indicate that by using proper calibrated parameters, good correlations between simulated and measured grades as well as dilution entry are obtained. The potential use of the numerical tool to evaluate drawpoint spacing in terms of recovery and dilution for block/panel caving operations is also presented.

Introduction

In block caving methods, the gravity flow characteristics of the ore and waste play an important role in determining ore recovery and dilution content. These characteristics should be established in advance, during the design stage of a block cave, in order to determine optimal spacing between drawpoints. Additionally they should be taken into account during mine operations in order to determine optimal draw control practices. To date, the knowledge of gravity flow of caved rock in block caving is based on physical modelling using sand as the tested material and a limited number of observations in the field[1,2].

Over the few last years, large physical modelling using gravel as the testing material has been conducted in order to understand the behaviour of coarse caved rock[3,4]. The experimental results have challenged some of the fundamentals of gravity flow used for determining the drawpoint spacing of block caves. For instance, interaction of extraction (IEZ) and movement zones (IMZ) occurred only when flow zones overlapped. These findings seem to contradict spacing rules derived from physical modelling using sand, which are the basis of current block caving design practice[2]. Despite the debate, there is consensus among practitioners and researchers about the need for conducting full-scale tests in this field to clarify some of the design criteria used in block caves.

As noted above, physical modelling has been the main tool used by researchers to gain understanding of the gravity flow pattern in block caving. This is due to the difficulties in observing the flow of the caved rock in the field. To date, there have been no full-scale tests conducted at mine scale in block caves using markers (to the extent of physical modelling) that could be confidently used to compare results and develop gravity flow hypothesis.

One of the possible disadvantages of using physical modelling is the distortion that may be induced in the phenomena under study by reducing the geometrical scale. Theoretical and experimental evidence have suggested that, at least in the free flowing stage, the gravity flow characteristics of fragmented material may be extrapolated between different scales. This is provided that overall geometrical similitude and similar materials friction angles are constant between systems[4].

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Together with the research conducted in the large physical model, the authors developed a numerical approach to gravity flow using the cellular automata in order to allow comparisons between full-scale and model-scale data. A second objective was to test the potential of the numerical tool for mine design. In this paper the development of this model is presented, including the mathematical background, calibration, comparisons with full-scale data and its potential use for mine design purposes.

Mathematical models of gravity flow

In the literature related to gravity flow in block caving, several mathematical models have been postulated. Some of them have been used in the mining industry while others have been developed for fundamental research. Mathematical approaches of gravity flow may be classified according to the fundamentals and purpose for which they have been developed in dynamic, Kinematic and ruled-based models. A synthesis of the main features and applications of the different approaches is presented in Table I[5].

To date, empirical methods, based on mixing templates, are the methods most widely used by the mining industry. Heslop and Laubscher's volumetric mixing methodology is used for mine planning, since it provides a rapid way of calculating vertical mixing as material is drawn through several draw points[1,6]. The methodology has been implemented in commercial software, such as PC-BC[7] and in some cases it is included in software developed by mining companies. Template mixing requires the height of interaction, the geometry of the flow zone and the dilution entry point as input flow parameters. These parameters may be found through back analysis or from gravity flow models.

Dynamic models emulate the gravity flow from a mechanical perspective, calculating forces between particles and from these, the movement or particle's kinematic[8,9]. These are the more fundamental approaches but are not amenable to full-scale validations. This is due to the limited number of particles that can be simulated using current computer capacity. As part of an international collaborative research effort in which this research was developed, a gravity flow model, known as Rebop, was developed to rapidly emulate the gravity flow using rules derived from PFC which is a discrete element model[10]. Rebop is a kinematic model in the sense that it uses a mass balance to establish adjustments to the flow zones (movement and extraction zones).

Mathematical models, which simulate the kinematics of the gravity flow, include other approaches such as stochastic[11–16] and cellular automata[17–19]. Some of the models listed in this paper have been developed for block caving applications whereas others have been developed for the flow in silos and bins. As can be seen in Table I, the authors have classified the kinematic models of flow in stochastic and cellular automata according to the use, or not, of local rules to emulate the flow. In the stochastic approach, the flow is simulated using discrete elements migrating downwards through the effect of gravity as material is removed at an exit point. This is simulated through a stochastic process as originally proposed by Litwiniszyn[20] and Mullins[21]. In the cellular automaton approach (CA), local rules, in addition to the set of probabilities, determine the movement of particles.

Despite the large amount of work spent on the development of numerical tools to describe the flow in block caving mines, there is still the need to develop flexible tools validated through experiments and full-scale data. A thorough review of CA and stochastic approaches for block caving indicated that past models are not able to simulate the movement zone as described in many experiments[5]. This

Table I

Summary of numerical models used to emulate the gravity flow of fragmented materials

Modelling approach	Contact forces	Discrete media	Gravity flow patterns	Input parameters	Calibration and validation	Ability to study flow mechanisms	Draw control applications
Distinct element[8,9]	✓	✓	✓	Material characteristics obtained though classical mechanics: particle size, shape and friction	Limited amount of elements limit the possibility of validation	✓	X
Finite element[22]	✓	X	X	Continuum material parameters: friction angle, elastic modulus, dilatancy and failure model	Particle size cannot be incorporated	✓	X
Rebop[10]	X	X	✓	Erosion, collapse and velocity profile	Can be calibrated using experimental IEZ and IMZ's	X	✓
Stochastic continuum[15–16]	X	X	✓	Diffusion coefficients, density change in a continuum media, boundary conditions	Can be calibrated using experimental IEZ and IMZ's	X	✓
Stochastic[11,14]	X	X	✓	Probabilities of fill vacancy	Can be calibrated using experimental IEZ	X	✓
Cellular automata[17,19]	X	✓	✓	Probabilities of fill vacancy and rules for transition function	Can be calibrated using experimental IEZ and IMZ's	✓	✓
LGrCA	✓	✓	✓	Modes of particle collision and propagation in a fixed lattice	Can be calibrated using experimental IEZ and IMZ's	✓	X
Volumetric models[6,7]	X	X	X	Flow parameters including PED, height of interaction and flow geometry	Could be calibrated using mine grades or a flow model	X	✓

paper presents the development of a new CA logic aimed at replicating the flow behaviour observed in a large physical model. A practical example of the use of the numerical model to determine drawpoint spacing for block and panel caving is also presented.

FlowSim: A new model for gravity flow based on cellular automata

There are several advantages to the development of a new numerical tool for the simulation of the gravity flow of caved rock using the cellular automata approach (CA). Through local rules, The CA could incorporate different boundary conditions, types of superficial flow and rates of fines migration, considering the discrete characteristics of the fragmented media. It could also be easily implemented using current PC capacity. The main disadvantage of CA is that the correct local rules are not easy to find and experience in the use of CA is required. FlowSim, which is the name given to the CA code developed in this research, is based on geometrical parameters and a transition function to simulate the flow in unconfined and confined conditions.

Geometrical parameters

A CA is a discretization of the caved rock into regular and rectangular blocks (or arrays) where explicit simulation of particles is not intended. Instead, the size of the blocks is related to the drawpoint dimensions being at least equal to or less than the drawpoint width. The blocks may have three different states: granular, void and solid. In general, the domain of a CA is expressed as:

$$\Gamma = \{c_{i,j,k}, i = 1..n_x, j = 1..n_y, k = 1..n_z; n_x, n_y, n_z \in \Re$$

$$c_{ijk} \begin{cases} 1 \\ 0 \\ -1 \end{cases}$$

where c_{ijk} is the state of a block located in the (i,j,k) position within the array, and has a value of 0, 1 or -1 depending on whether the block is granular, void or solid respectively. These blocks may change their state from granular to void or vice versa. Changes from solid to granular may be used to simulate particular situations such as preferential cave direction or brow wear. In comparison to finite difference methods, this simplifies the setting of boundary conditions, to a more rational approach. Other attributes that may be included are changes in fragmentation and fines migration.

Regarding the features included in FlowSim, we opted initially to implement a simple model which could emulate the kinematic of the gravitational flow based on observations in the large physical model. Thus, a fines migration logic was not developed in the current approach as experiments using markers of different particles sizes in the physical model indicated that the fines do not percolate at the rate of previously believed. Results indicated that the fines fraction, which was characterized through markers having a 1/10 of the size of the mean particle size, moved an average an 11% faster than the modal size. Moreover, the percolation rate decreases to zero with the height of draw[5].

In FlowSim the gravitational flow is simulated as a stochastic process of particles moving downwards as new vacancies are created through the extraction of material at a drawpoint. This is done using a Monte Carlo technique and assuming a probability set that is inversely proportional to distance from a void. Thus, if considering symmetry and gravity flow, a void has a chance of moving, in a 9-block neighbourhood, to a higher position (p_a), higher and to the side (p_l) or higher and diagonally (p_c) as shown in Figure 1. This means that there are three probabilities to be calculated, which according to distance are:

$$p_a = \frac{d_{v1}^{-n}}{d_{v1}^{-n} + 4d_{v2}^{-n} + 4d_{v3}^{-n}}, p_l = \frac{d_{v2}^{-n}}{d_{v1}^{-n} + 4d_{v2}^{-n} + 4d_{v3}^{-n}},$$

$$p_c = \frac{d_{v3}^{-n}}{d_{v1}^{-n} + 4d_{v2}^{-n} + 4d_{v3}^{-n}},$$

where d_{vi} (i = 1..3) are the distances between the void and the central, lateral and corner cells respectively, and n is an adjustable constant. Thus, given a certain block size and n value, there is a unique set of probabilities that may be used, reducing the number of parameters to three (the three dimensions of the cells). When flow is not gravitational the probabilities may be adjusted accordingly.

Modelling confined granular flow

Confined granular flow refers to the movement of particles generated by the space created by other in-contact particles. According to this definition, there are many situations that may be classified as confined granular flow, including the flow in a silo, flow in ore passes, flow of blasted rock in sublevel caving and flow of caved rock in a block caving. Experimental observations have shown that when material is discharged through a drawpoint, the flow zone develops in a finite volume characterized by a low porosity zone[23]. This unique characteristic of fragmented rock has been explained in terms of the ability of the granular material to arch at the top of the flow zone[5,9].

FlowSim simulates this arching by setting a micro-arch rule which repeats over the CA domain. This is carried out in the following way: for a given number of voids in a volume, the flow can continue only if there are enough voids to break the arch abutments. In the case that there are not enough voids, this logic results in the formation of a stable structure of low density which defines the flow zone. This is based on

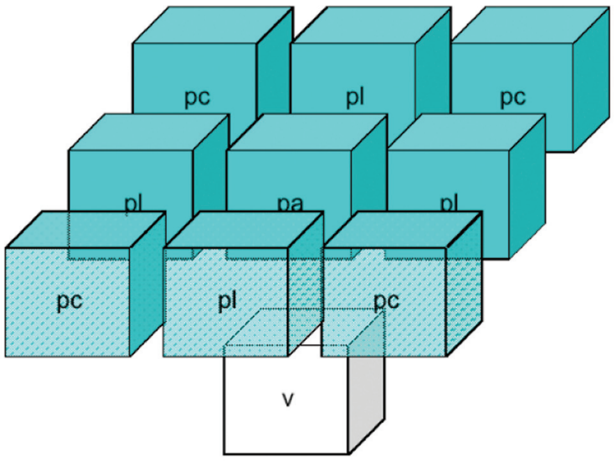

Figure 1—Flow developed using the vacancy approach: p_a, p_l and p_c are the central, lateral and corner probabilities

the fact that granular materials need to dilate in order to flow under shear[15]. This 'extra space' is provided by the drawing of material at the drawpoint, which induces swelling or dilation in the granular mass. In FlowSim this is modelled by setting a minimum required number of voids to break the arch in the cell neighbourhood ($c_v^{threshold}$). The number of voids c_v is calculated for each void- block as (see Figure 1):

$$c_v = \sum_{k=0}^{1}\sum_{j=-1}^{1}\sum_{i=-1}^{1} c_{ijk}, \forall c_{ijk} > 0$$

where c_{ijk} =1 if there is a void in the ijk coordinate and 0 if not. Thus, flow proceeds only if:

$$c_v >= c_v^{threshold}$$

Modelling of surface flow

Surface or unconfined flow refers to the movement of a particulate material as it tries to reach its angle of repose. In FlowSim this is modelled using the self-organized criticality approach[24], modified to simulate flow in three dimensions. The height of a pile or column located at coordinates (i,j) is calculated by adding the number of granular blocks from a regional valley in a certain domain. The height of the pile h_{ij} and the height difference in a given direction j^* are calculated for each of the columns i,j:

$$h_{i,j} = \sum_{k=1}^{n_z} c_{ij,k}$$

$$\Delta h_{i,j} = h_{i,j+1} - h_{i,j}$$

The flow would occur if the difference between the heights of adjacent piles reaches a certain limit, or whenever $\Delta h_{i,j} > dz$ where dz is a threshold taking the local angle of repose into account. This is related to the horizontal dimensions of the cells dx and the angle of repose α by:

$$dz = dx \tan\alpha$$

To simulate surface flow in three dimensions, this process is carried out for various j directions in a random manner until the system reaches a kinematic equilibrium.

Calibration and results of FlowSim

The above model was written in the Matlab software and results were displayed in coloured layers as used by researches studying the gravity flow in physical models. As noted in Figure 2, results indicate that flow zones using the above algorithm have an ellipse shape in accordance with experimental evidence.

As a first stage, the model was calibrated and validated using experimental results obtained from the physical model program[5].It was concluded that the mathematical model could be used for quantitative purposes when calibrated.

Comparisons to full-scale data

Once *FlowSim* had been calibrated using the large physical model, a study was conducted aimed at comparing the results using mine data. This was carried out using draw control data obtained from the operations at the Inca Norte sector of the El Salvador mine and the Esmeralda sector of El Teniente mine, both divisions operated by Codelco, Chile, using the panel caving method.

Inca Oeste Norte—El Salvador mine

El Salvador mine is a unique case study for carrying out comparisons of gravity flow on a large scale, given the characteristics of the diluting material. This is composed of a lixiviated waste (limonite) which is also used as a geological marker due to its distinct red colour. This allows mine operators to identify the tonnage and percentage of dilution as the draw continues. Moreover the percentage of observed limonite is also used as a criterion to determine the amount of dilution and drawpoint shut-off. In the El Salvador Mine there are strict draw control practices which have derived from the construction of a well maintained database containing the *in situ* block model, the amount of tonnage extracted per drawpoint, measured copper grades and the amount of observed dilution per drawpoint at different stages of draw.

The sector selected in this study was the Inca Norte which extracted what in local Chilean terms is called primary ore. The primary ore in Inca Norte was mainly composed of a rock mass containing porphyry and andesites which had a RMR ranging from IIIA to IIA. In Lausbcher's classification system this corresponds to the extraction of competent to very competent rock mass[25]. The Inca Norte sector is characterized for having *in situ* ore columns ranging from 130 to 140 metres in height defining an ore-dilution contact which is

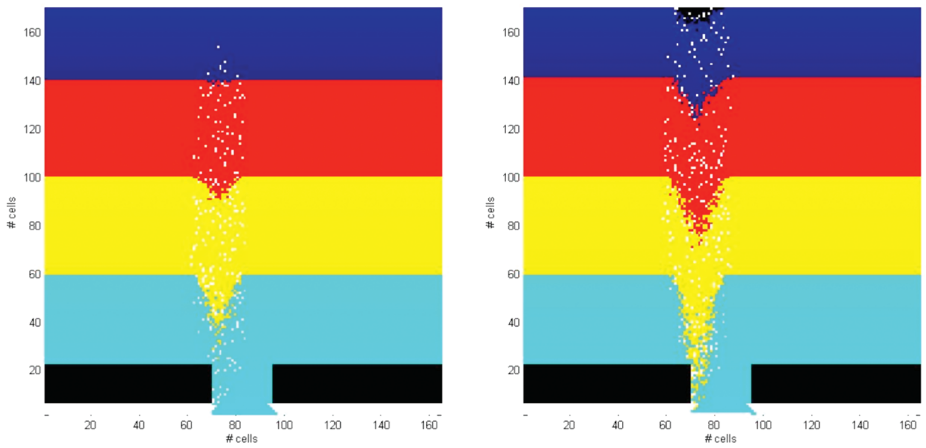

Figure 2—Examples of FlowSim for a single drawpoint at two different stages of draw

almost entirely horizontal. Adjacent to the south was the Inca-Central sector which was had been previously extracted. This meant that the ore-diluting interface was sub vertical.

For the validation of FlowSim a series of drawpoints were selected to form clusters of points at different locations of the mine as indicated in Figure 3. These clusters were selected to simulate a three dimensional configuration of drawpoints. These drawpoints were classified according to: the existence or not of other drawpoints in boundary points or drawpoints without a point next to them; points close to an orepass; and interior points or drawpoints surrounded by other drawpoints.

A comparative analysis was carried out using as an index the dilution entry point (PED) determined as the ratio between the tonnage corresponding to the *in situ* ore column and the tonnage extracted where dilution is observed at a drawpoint. This analysis was carried out for each of the drawpoints and the errors were calculated accordingly. The mean error for estimating the PED as noted in Figure 4 was 26%. While this error may be considered high, the PED in the mines reaches similar dispersion, even for drawpoints with apparently similar geometry and draw conditions.

A statistical analysis of all drawpoints analysed is presented in Table II. This considers 55 drawpoints equivalent to an area of 12 375 m². The results indicated that, given the dispersion in the PED in the mine, the current evidence suggests that the model may be used to predict dilution entry.

Table II

Statistical analysis of simulated and actual PED in Inca Norte Mine

Variable	PED simulated	PED observed
Mean [%]	72.6	73.2
Standard deviation [%]	27.9	37.3
Number of data	55	55
Pearson's correlation coefficient	0.73	
Degrees of freedom	54	
t-statistic	-0.17	
P(T<=t) two-sided	0.87	
Critical t-statistic for rejecting equal means	2.0	

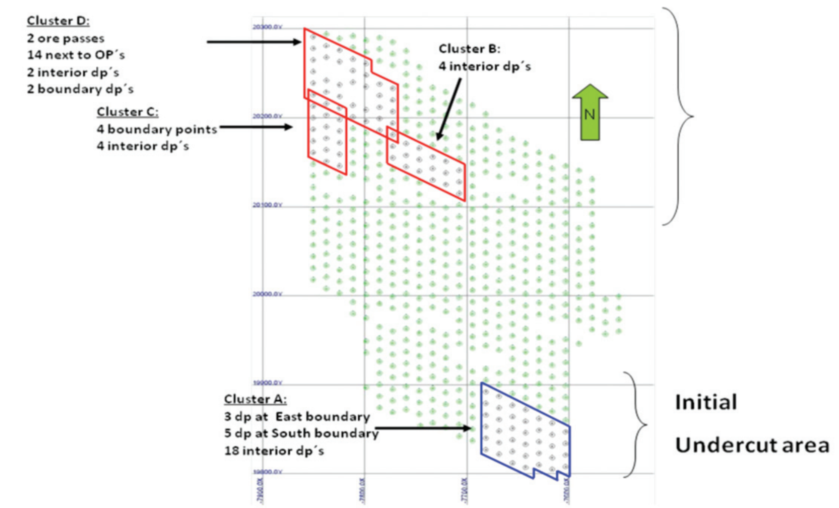

Figure 3—Schematic view of the Inca Norte Mine showing the clusters selected for analysis of FlowSim

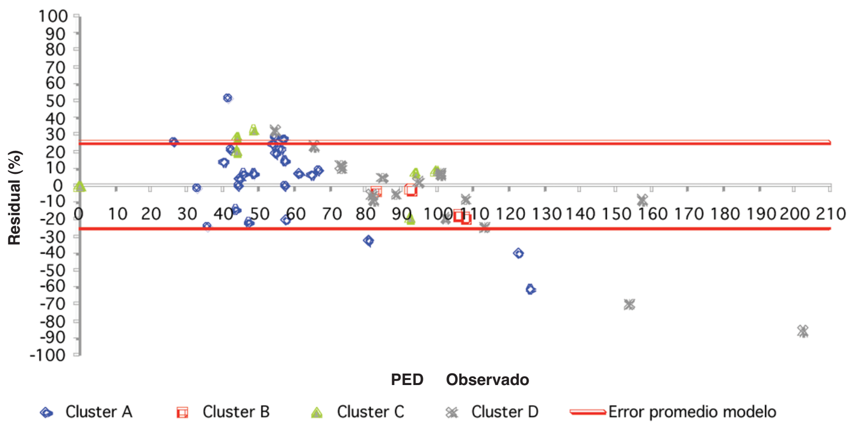

Figure 4—Average model error for the different clusters as a function of the PED

Esmeralda sector—El Teniente mine

The Esmeralda is one of the mines currently in operation at the El Teniente complex and to date is the sector with the highest production. The height of the *in situ* ore reaches a mean of 150 m with an RMRL ranging from 53 to 68. This has resulted in coarse fragmentation with a 20% over $2m^{3}$[26].

Five clusters of drawpoints were arbitrarily selected from the sector as indicated in Figure 5, which shows an overview of the footprint. In the case of the Esmeralda mine there is information regarding extracted tonnages, *in situ* (block model) and extracted copper grades measured at the drawpoint on a shift to shift basis. There are no geological markers as in the case of El Salvador mine. Regarding the validation of flow models, grades are not as good as geological or physical markers, because the *in situ* and extracted grades are estimates of the actual grades. Despite this, comparisons were carried out considering the case of mines that may have only grades, and therefore would require a methodology for calibration and validation for a flow model.

Table III presents the results of the mean relative error on a shift to shift basis between the simulation and the actual grades for each drawpoint. The results indicate that the errors may reach up to 47% but decrease to about 10% when averaged to four drawpoints (cluster level). This result shows that the simulator is able to replicate the grades within a 10% interval for most of the clusters analysed.

FlowSim: a tool for determining drawpoint spacing for block caving

Once the numerical model seemed to reproduce, the gravity flow behaviour in block caving mines within the limits of the database considered here, a methodology was developed to demonstrate the potential of FlowSim as a tool for the assessment of drawpoint spacing in a production level design for block/panel caving applications.

The methodology developed in this research considered simulations of the flow model in a basic production unit. The drawpoints were located in a El Teniente's configuration (Figure 6). Simulations were conducted varying the distance between production drifts (Dc) and drawpoint spacing (De) under concurrent draw. From these the percentage at which dilution would appear at a drawpoint, as well as the mixing and recovery were calculated.

Table III

Relative errors between actual and simulated grades

Cluster	Drawpoint	Estimative error (%)	Standard deviation (%)	Overall cluster error (%)
1	2710H	-47.07	10.21	-17.53
	2710F	-28.03	8.10	
	2910H	1.09	3.89	
	2910F	3.90	3.17	
2	1112H	-16.06	6.62	-15.82
	1112F	-5.90	12.14	
	1312H	-28.29	7.56	
	1312F	-13.03	14.69	
3	1104H	-0.88	5.89	-9.59
	1104F	0.49	5.31	
	1304H	-18.88	7.49	
	1304F	-19.10	6.22	
4	2914H	9.12	6.99	-1.13
	2914F	0.19	4.87	
	3114H	-1.97	5.56	
	3114F	-11.85	4.88	
5	1706H	-3.49	7.13	-1.85
	1706F	-14.76	5.52	
	1906H	1.66	9.27	
	1906F	9.16	5.26	

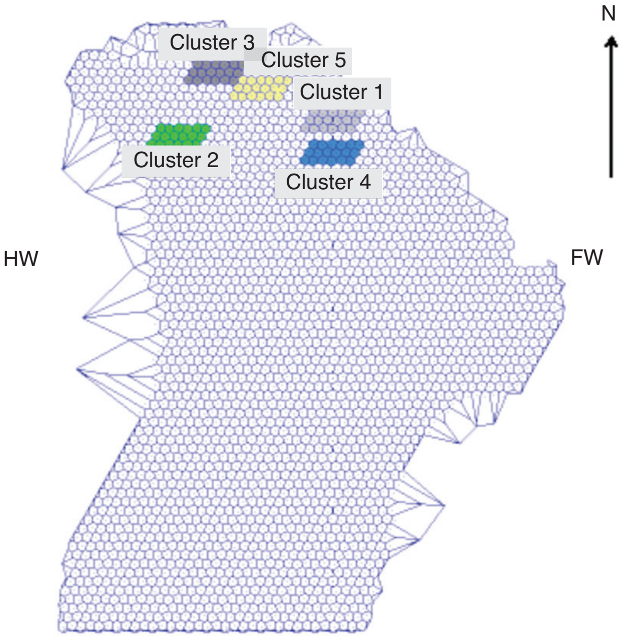

Figure 5—Schematic view of draw points at the Esmeralda sector showing the clusters selected for *FlowSim* analysis

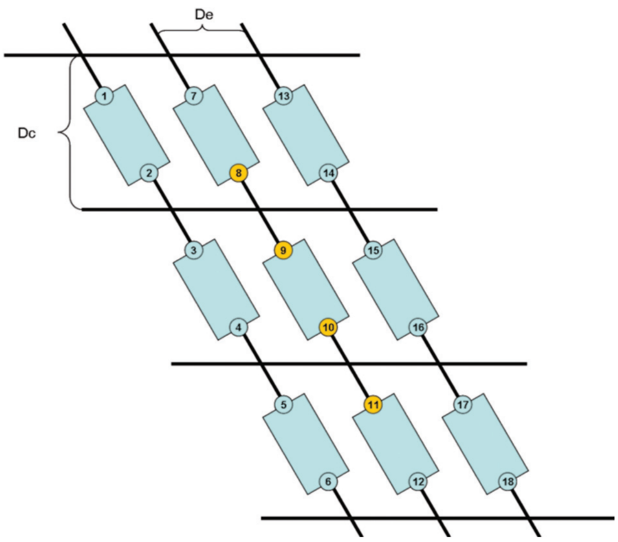

Figure 6—Simulated PED for 150 m ore column as a function of production and drawpoint spacing

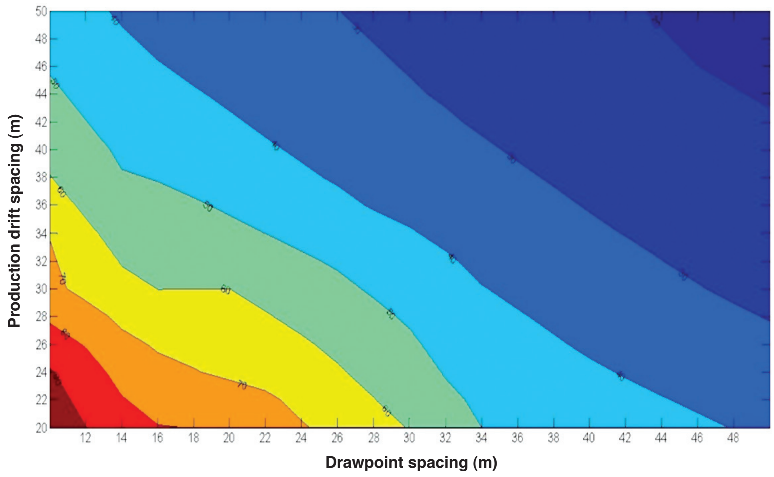

Figure 6—Simulated PED for 150 m ore column as a function of production and drawpoint spacing

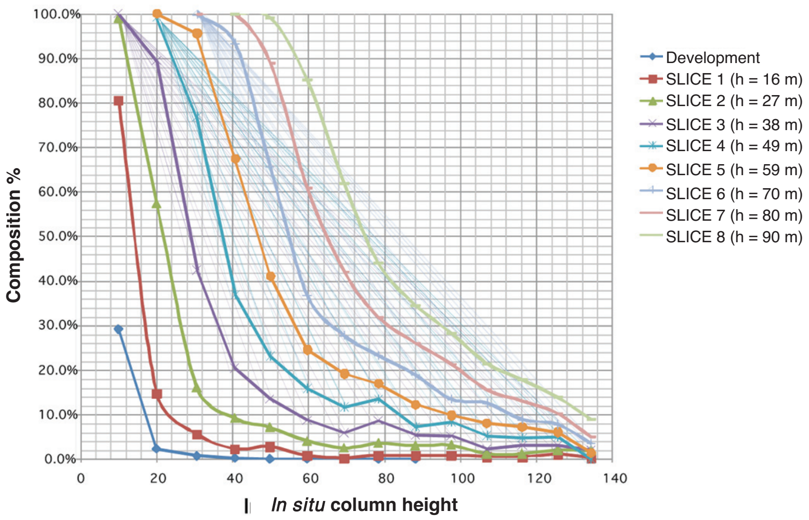

Figure 7—Mixing chart for a 300 m ore column in a 30 x 15 m layout

From the simulations, design charts indicating the PED for various drawpoint configurations were developed as presented in Figure 7 which shows results for a 150 m ore column. These results indicate that dilution entry decreases as the drawpoint or production level drift spacing increases. Similarly, other numerical results using different draw column heights indicate that dilution entry decreases as column height decreases. These results seem to concur qualitatively with what has been observed in block caving practice[24].

The corresponding mixing and recovery curves, were also calculated (Figure 7 and 8). These curves correspond to simulations of a production level layout of 30 x 15 m, an ore column of 150 m and an ore-waste interface located at 90 m from the back of production drift (equivalent to slice number 8 and above). The composition curves (Figure 7) show a nonlinear correlation with an extracted mass or column height equivalent, which was noted in the construction of the volumetric model. The equivalent recovery curves (Figure 8) are presented showing the recovered ore as a function of the extracted *in situ* height.

From these curves it is possible to estimate the total recovery for a given drawpoint shut-off strategy and ore grades for a drawpoint. For example, we might consider a diluting material of zero grade, an ore grade of 1% copper distributed uniformly along the *in situ* column, and a cut-off grade strategy of 0.6% copper. Thus, the extraction of the drawpoint would be uneconomical when 40% of the total material observed at the drawpoint is reported as dilution. From Figure 7 this scenario would be achieved when extracting the tonnage corresponding to a column height of 70 metres. The recovery associated with this column height is calculated by adding the recovered percentage per *in situ* slice as presented in Figure 8. In this case the total recovery reaches 74% of the total *in situ* column.

Conclusions and discussions

Over the few last years significant new efforts have been made investigating the gravity flow of caved rock. This paper shows the development of FlowSim, a numerical approach based on cellular automata, to emulate the gravity flow of the

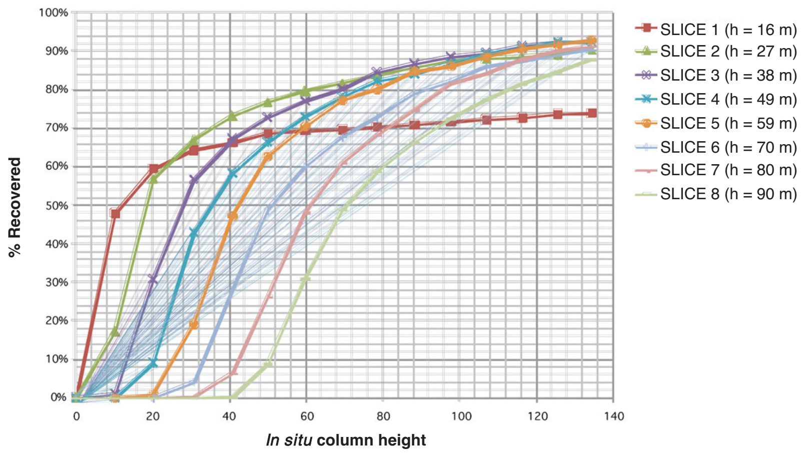

Figure 8—Recovery as a function of equivalent extracted height for condition shown in Figure 7

caved rock. The results indicate that when properly calibrated, CA may be used as a predictable tool which may help in the design of block cave production levels.

There is still a long way to go in developing tools and models that incorporate all the gravity flow characteristics observed in a block cave, such as compaction, fines migration and secondary breakage. It seems that at this stage of knowledge, simple models may emulate the flow of the caved rock at least within the ranges of certainty that exist in the data collected by the mines. There is still the need to conduct full-scale tests which would allow the development of new theories and concepts.

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