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PIONEERING PROGRESS :
THE FUTURE OF ROCK ENGINEERING

Key block theory for the miner: a novel simplified method for determining block removability

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Contents

Recognition

Goodman, R.E. and Shi, G., 1985.
Block Theory and its application to Rock Engineering.

Yan, J., Chen, J., Zhang, Y., Liu, Y., Zhao, X., Xue, J., Zhu, C., Mehmood, Q. and Wang, Q., 2023.
Semi-automatic extraction of dangerous rock blocks from jointed rock exposures based on a discontinuity trace map.

Overview of key block theory

- Rock mass is not a continuum
- Discontinuities divide the rock mass into interlocked discrete blocks
- Rock mass is stable until excavation
- Excavation creates voids
- Discrete blocks become free to move into the voids
- Movement of a “key block” results in a cascade effect
- Determining removability of a block – Foundation of key block theory

Mathematical perspective:

- Joints are approximated to 3D planes:
- Planes intersect to form discrete block
- Vector analysis used to judge if a block is removable

Vector analysis (Edge vector method):

$$A_{J1}x + B_{J1}y + C_{J1}z = 0$$
$$A_{J2}x + B_{J2}y + C_{J2}z = 0$$
$$A_{J3}x + B_{J3}y + C_{J3}z = 0$$

Planar equations

$$A = \sin(\alpha).\sin(\beta)$$
$$B = \sin(\alpha).\cos(\beta)$$
$$C = \cos(\alpha)$$

Unit vector equations

$$\hat{n}_{J1} = -(A_{J1}, B_{J1}, C_{J1}) \quad \hat{n}_{J1} = (A_{J1}, B_{J1}, C_{J1})$$
$$\hat{n}_{J2} = -(A_{J2}, B_{J2}, C_{J2}) \quad \hat{n}_{J2} = (A_{J2}, B_{J2}, C_{J2})$$
$$\hat{n}_{J3} = -(A_{J3}, B_{J3}, C_{J3}) \quad \hat{n}_{J3} = (A_{J3}, B_{J3}, C_{J3})$$

Inward normal vectors

$$I_{J1J2} = \hat{n}_{J1} \times \hat{n}_{J2}$$

$$I_{J1J2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_{J1} & B_{J1} & C_{J1} \\ A_{J2} & B_{J2} & C_{J2} \end{vmatrix}$$

$$I_{J1J2} = \begin{vmatrix} B_{J1} & C_{J1} \\ B_{J2} & C_{J2} \end{vmatrix}\hat{i} - \begin{vmatrix} A_{J1} & C_{J1} \\ A_{J2} & C_{J2} \end{vmatrix}\hat{j} + \begin{vmatrix} A_{J1} & B_{J1} \\ A_{J2} & B_{J2} \end{vmatrix}\hat{k}$$

$$I_{J1J2} = (B_{J1}C_{J2} - C_{J1}B_{J2}, C_{J1}A_{J2} - A_{J1}C_{J2}, A_{J1}B_{J2} - B_{J1}A_{J2})$$

Intersection vectors: Cross product

Scalar products: Dot product

$$\hat{n}_{J1}.I_{J1J2} \quad \hat{n}_{J1}.I_{J2J3} \quad \hat{n}_{J1}.I_{J1J3}$$
$$\hat{n}_{J2}.I_{J1J2} \quad \hat{n}_{J2}.I_{J2J3} \quad \hat{n}_{J2}.I_{J1J3}$$
$$\hat{n}_{J3}.I_{J1J2} \quad \hat{n}_{J3}.I_{J2J3} \quad \hat{n}_{J3}.I_{J1J3}$$

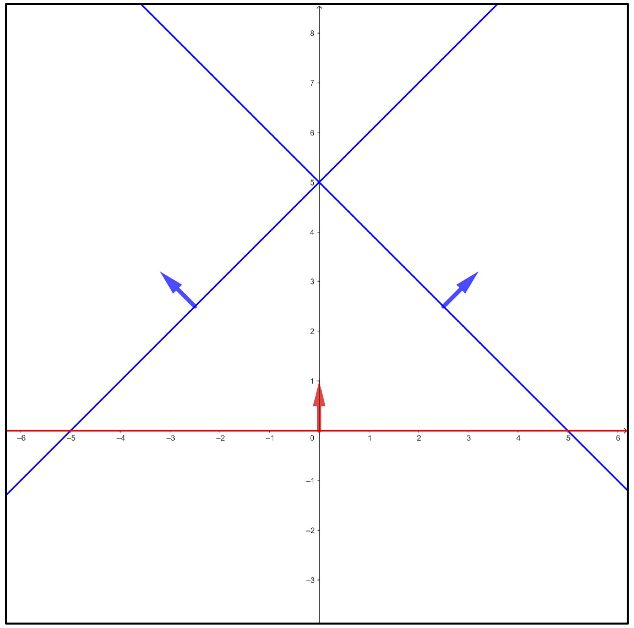
Gravitational removability number

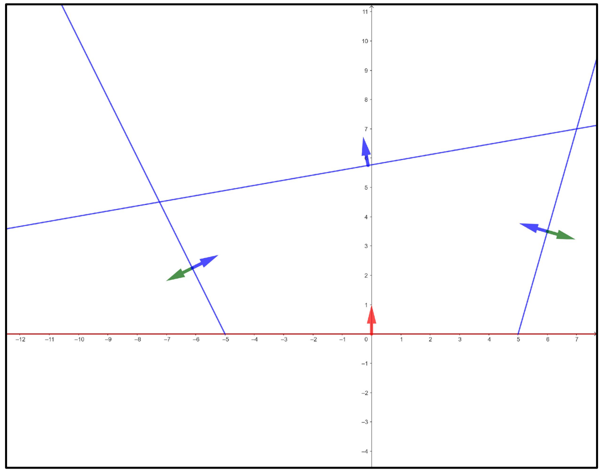
Fundamental principle: *A block can only move if, firstly, it exists, and secondly, if it is free to do so.*

Principles of GRN:

- Gravity drives block movement
- A block consists of intersecting discontinuities
- Every discontinuity either constrains movement or contributes to it
- A block is removable if the contributing discontinuities outweigh the constraining discontinuities

Rock mass vector (RMV): Vector pointing into the rock mass





$$\cos\theta = \frac{a \cdot b}{|a||b|}$$

Angle between vectors

The relationship between the upward normal vector and the RMV is determined by which half-space of the joint the block is located

Constrain: If the angle between the RMV of the free surface and that of the joint is more than 90°

Contribute: If the angle between the RMV of the free surface and that of the joint is less than 90°

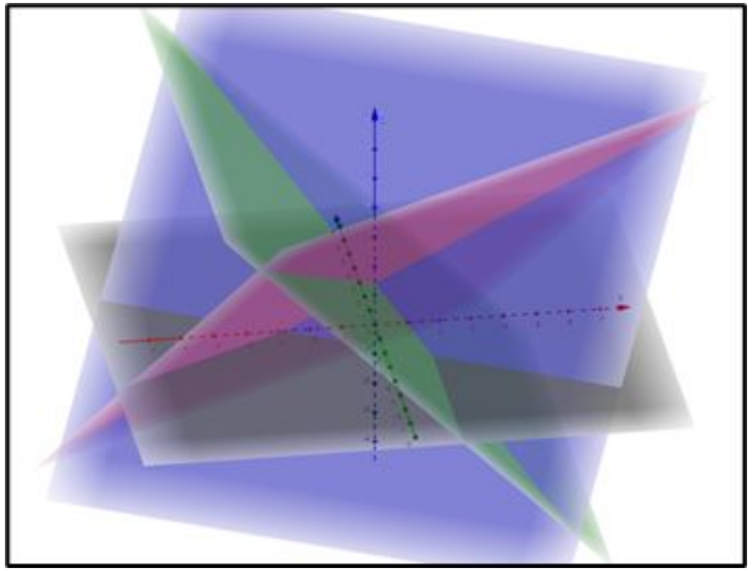
RC: Total constraint – Sum of absolute z-values of the constraining joint RMVs

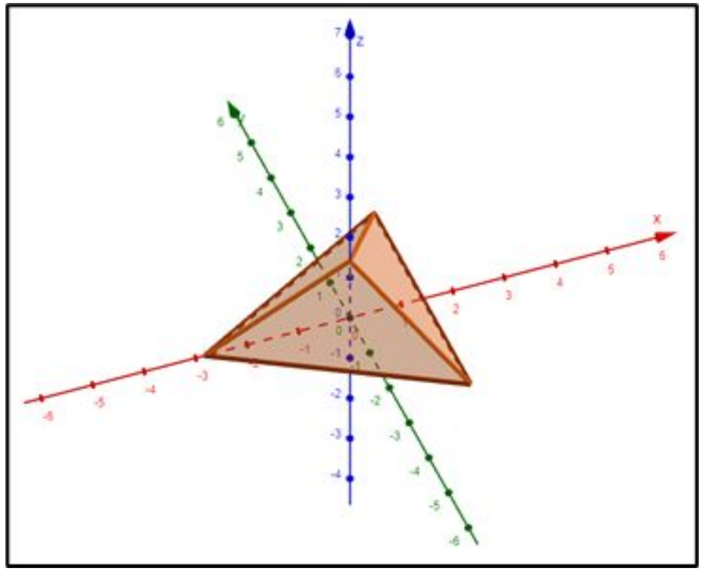
RG: Total contribution – Sum of absolute z-values of contributing joint RMVs

$$\mathrm{GRN} = (1 + \mathrm{RC}) - \mathrm{RG}$$

If GRN > 0: Block is not removable
If GRN < 0: Block is removable

Example

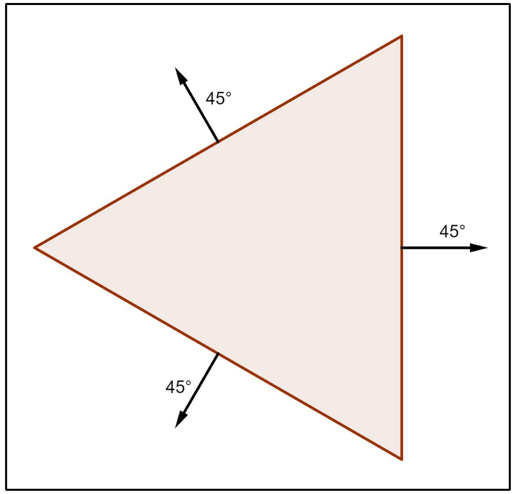




ID	α	β	½ Space
FS	0	90	U
J1	45	90	L
J2	45	330	L
J3	45	210	L

GRN Calculator

Input				Upwards unit normal			RMV					
ID	α	β	½ Space	A	B	C	x	y	z	Dot	Contribute	Constrain
FS	0	90	U	0	0	1	0	0	1			
J1	45	90	L	0.707107	4.33E-17	0.707107	0.707107	4.33E-17	0.707107	0.707107	0.7071068	0
J2	45	330	L	-0.35355	0.612372	0.707107	-0.35355	0.612372	0.707107	0.707107	0.7071068	0
J3	45	210	L	-0.35355	-0.61237	0.707107	-0.35355	-0.61237	0.707107	0.707107	0.7071068	0
Removable											2.1213203	0
											GRN	-2.12132



Geometric conditions

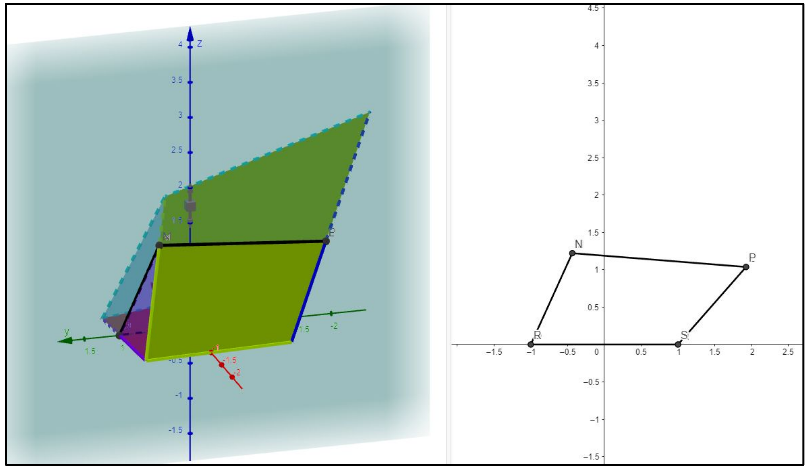
Base Comparison			
Number of constituent joints	3	4	5
Base Statistics			
Blocks generated	100,000	100,000	100,000
Removable according to EV	47,417	15,988	4,184
Non-removable according to EV	52,583	84,012	95,816
EV & GRN removable			
Count	42,789	15,480	4,179
Agreement	90%	97%	99.88%
% of total blocks	43%	15%	4%
EV & GRN non-removable			
Count	45,332	49,405	49,927
Agreement	86%	59%	52%
% of total blocks	45%	49%	50%
EV removable / GRN non-removable			
Count	4,628	508	5
Disagreement	10%	3%	0.12%
% of total blocks	5%	1%	0%
EV non-removable / GRN removable			
Count	7,251	34,607	45,889
Disagreement	14%	41%	48%
% of total blocks	7%	35%	46%

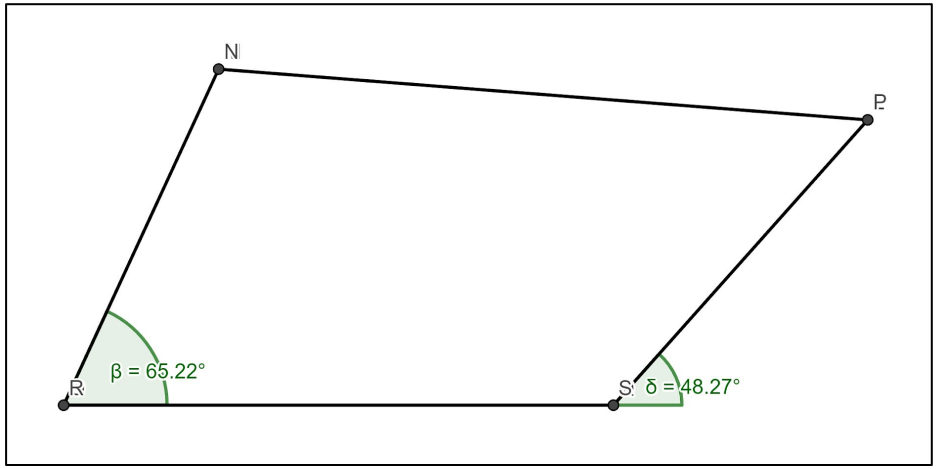
Experimental results

- Fair correlation on removability, increases with block sides
- Less satisfactory correlation on non-removability

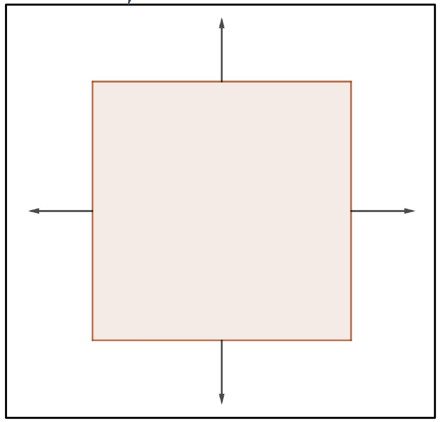
But why?

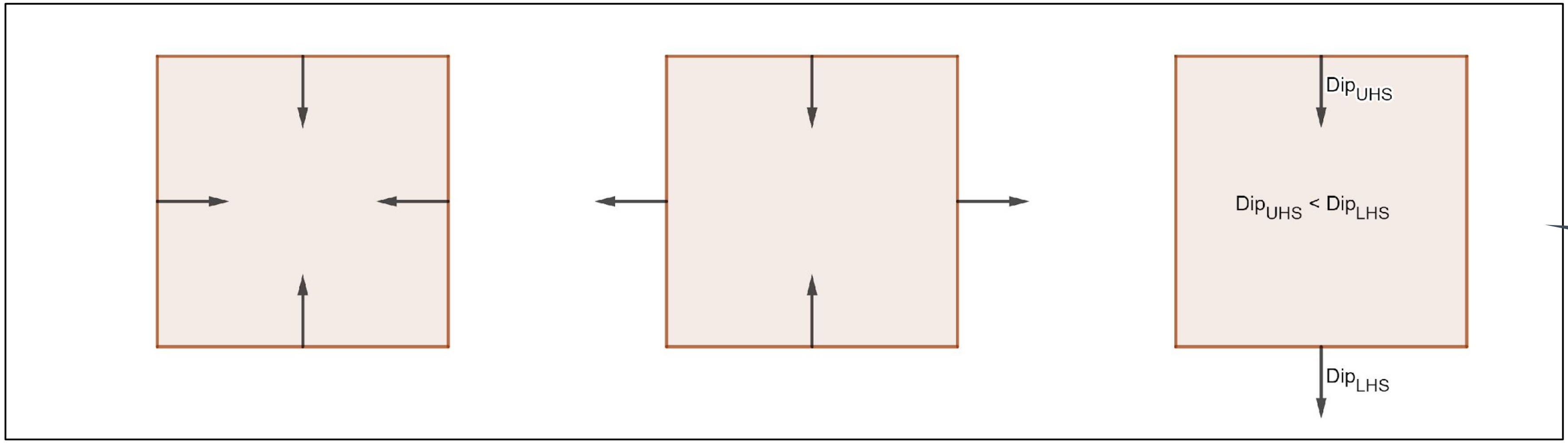
Outright removability / non-removability - Geometry





Outright removable





Outright non-removable

Result

Calibrated Balance Sheet					
Number of constituent joints			**3**	**4**	**5**
Number of blocks generated			**100,000**	**100,000**	**100,000**
Geometric calibration					
1.	Outright removable	All LHS	(633)	(1,554)	(1,558)
2.	Opposite constrained	UHS-UHS	0	(7,112)	(18,834)
3.	Constrained by collision	α-UHS < α-LHS	0	(17,430)	(20,236)
4.	Potential rotation	At least 1 UHS	(11,246)	(9,019)	(5,266)
Subtotal			**88,121**	**64,885**	**54,106**
EV & GRN removable			(42,789)	(15,480)	(4,179)
EV & GRN non-removable			(45,332)	(49,405)	(49,927)
Blocks not accounted			**0**	**0**	**0**

Conclusion

Experimental results

- Fair initial correlation on removability, increases with block sides
- Less satisfactory correlation on non-removability
- Application of geometric filter and identification of potential rotation lead to perfect agreement with EV

Further work

- Closer look at potential rotation and correlation to GRN
- Practical application of geometric filters for everyday use

THANK YOU

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